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SMITHSONIAN ASTROPHYSICAL OBSERVATORY

PROGRAM WRITEUP (SCROGE)

by

J. R. Cherniack and E. M. Gaposchkin

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CR-50,797

SMITHSONIAN ASTROPHYSICAL OBSERVATORY  
PROGRAM WRITEUP (SCROGE)<sup>1</sup>

J. R. Cherniack<sup>2</sup> and E. M. Gaposchkin<sup>3</sup>

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The method employed I would gladly explain  
While I have it so clear in my head,  
If I had but the time and you had but the brain--  
But much yet remains to be said.

--Lewis Carroll, "The Hunting  
of the Snark"

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With the advent of artificial earth satellites and their usefulness in increasing our knowledge of the earth we live on, a complex problem presented itself--that of tracking a satellite. Tracking a satellite with sufficient accuracy to be of scientific value has presented technologists in several fields with many interesting problems. Older astronomical techniques, though providing a sound basis, are not completely applicable.

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<sup>1</sup>This work was supported in part by Grant NSG 87-60 from the National Aeronautics and Space Administration.

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A satellite can complete one revolution in 1.5 to 2 hours; hence speed in the tracking operation is a very important factor. To "keep up" with the satellite, the observed positions must be processed through the computer within hours of the passage, and predictions must be available at the observing stations within hours of their calculation. The whole situation is made difficult because of the real time aspect of the tracking problem.

In order to obtain optical observations to an accuracy of a few seconds of arc, a special camera (the Baker-Nunn camera) was designed and built; 12 camera stations were established around the earth; appropriate operational procedures were set up; a teletype communication network was organized; and digital computer programs were written to analyze the observations and provide ephemerides. The purpose of this special report is to describe one of the computer programs now used at the Smithsonian Astrophysical Observatory.

The tracking of satellites is a cyclical or regenerative process consisting of 3 consecutive steps:

- 1) An improved orbit is computed from recent observations or, if the satellite has just been launched, from the best estimate of the orbital parameters.<sup>4</sup>
- 2) Predicted positions are computed from this orbit and sent to the observing stations.
- 3) Observations are made from the predicted positions and returned. The observations of step 3 are then used in step 1. SCROGE,<sup>5</sup> a computer program written for the IBM 7090, fulfills step 2 of this cycle for the Baker-Nunn camera network. Based on an idea originated by George Veis and Jack Slowey, it is a combined effort of many members of the SAO staff. The output of the program is an operating schedule, the 666 message, sent to all stations, which lists the camera settings.

The Baker-Nunn camera was built specifically for optical satellite tracking. It is a triaxially mounted camera that can track along one axis. For a more complete description of the Baker-Nunn camera see Henize (1957). The camera tracks only in camera-centered great circles and at a constant tracking velocity. The complicating fact is that, in principle, a satellite's motion is neither in a great-circle arc nor at a constant angular velocity. In practice, however, the satellite's

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<sup>4</sup> This computation is carried out by the Smithsonian Differential Orbit Improvement program, referred to hereafter as DOI.

<sup>5</sup> SCROGE denotes Smithsonian Computations Relating Orbital Glimpses Everywhere.

motion does conform to great-circle arcs at constant angular velocity for short periods of time.

SCROGE does three things:

1. It determines when and from where the satellite is visible.
2. It computes the arcs along which the Baker-Nunn cameras will track to match the apparent motion of the satellite.
3. It produces an operating or observing schedule and camera settings for the station (the 666 message).

The program is organized in two parts, referred to as Phase 1 and Phase 2. Phase 1 combines operations 1 and 2, and Phase 2 performs the third. Phase 1 can be further broken down into the following steps:

- 1) input of station information;
- 2) input of satellite information;
- 3) computing the sun and satellite positions;
- 4) determining visibility;
- 5) computing, with visible arcs, the great circle arcs.

Steps 3, 4, and 5 are made for each satellite-station combination for the whole interval of interest. SAO will usually compute predictions for seven days in advance. However, this period is dependent on such other factors as the accuracy of the orbit and the operational requirements of the communications network, rather than on the program.

#### Phase 1

The satellite's motion is characterized by its position at discrete and equal intervals in time. The period for which predictions are desired and the increment in time are input parameters for each satellite.

For each time in the interval (taken in increasing order):

- 1) The satellite's position is computed.
- 2) For each station at this time:
  - a) The local position of the satellite is computed and, if the satellite is visible, the coordinates of the satellite and station are retained for future use.

- b) If this position completes an entire transit over the station, the retained points are used to fit (by the method of least squares) the best great-circle arcs that approximate to the satellite's motion with respect to the station. These great-circle arcs are accumulated for use in Phase 2.

The above procedure is adopted for each satellite.

## Phase 2

Phase 2 is essentially a bookkeeping program. It sorts all of the predicted observations by station, and within each station by time. It then outputs the observation in the 666 message format. Phase 2 computes any quantities that are of interest to the station personnel but are not necessary to the computations done in Phase 1. Phase 2 also ascertains whether its schedule to the station is realistic; in other words, it allows an appropriate length of time between observations to reset the camera. It deletes conflicting observations generally by trimming or reducing both of the observations.

We shall now describe the coordinate systems, the expressions used to evaluate satellite and sun coordinates, the visibility criteria, and the mathematics involved in computing the great-circle arcs that approximate the satellite's apparent motion.

## Coordinate systems

### 1) Inertial Geocentric (IG):

$x_1$ , Aries (1950.0),  
 $x_3$ , North Celestial Pole,  
 $x_2$ , chosen to make a left-handed system.  
Origin, center of earth.

N.B. The satellite orbit is referred to the system defined by  $x_1x_3$ , but with  $x_2$  chosen to make a right-handed system.

### 2) Rotating Geocentric (RG):

$y_1$ , Greenwich,  
 $y_3$ , North Celestial Pole,  
 $y_2$ , chosen to make a left-handed system.  
Origin, center of earth.

N.B. The station coordinates are fixed in this system.

The IG and RG systems are related by the sidereal angle (or sidereal time).

3) Local Station (LS) (one system for each station):

$z_1$ , east,  
 $z_2$ , north,  
 $z_3$ , zenith.  
Origin, station position.

#### Sidereal angle, sun and satellite positions

1) The instantaneous sidereal angle ( $\theta$ ) is computed by subroutine SIDTIM:

$$\theta = a + b(T - T_0) ,$$

where

$T$  = time in modified Julian Days, which are defined as the Julian Days minus 2,400,000.5,

$T_0 = 33,282$  (1 Jan 1950),

$a = .277987616$  (sidereal angle at 1 Jan 1950 minus nutation in right ascension in revolutions),

$b = 1.00273781191$  (Sterne [1960] gives this for revolutions per mean solar day for 1900.0).

2) The instantaneous position of the sun ( $\bar{S}_\odot$ ) is computed by subroutine GETSUN (vectors are denoted by a bar over the symbol). We determine the eccentric anomaly  $E_\odot$  from the first approximation formulas:

$$M_\odot = c + d(T - \tau_\odot), \text{ when } M_\odot \text{ is the mean anomaly.}$$

Whence

$$E_\odot = M_\odot + e \sin M_\odot ,$$

where

$T$  = the time as defined above,

$\tau_0 = 36,203,$  (0 Jan 1958)

$c = -.0531695,$  (mean anomaly at 0 Jan 1958)

$d = .017202125,$  (mean motion of sun)

$e = .01672664$  (eccentricity of sun).

We define:

$$\bar{E}_\odot = \begin{pmatrix} \cos E_\odot \\ \sin E_\odot \\ 1 \end{pmatrix},$$

$$A = \begin{pmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$B = \begin{pmatrix} .31317 \times 10^5 & 1.46163 \times 10^5 & -.523848 \times 10^3 \\ -1.34114 \times 10^5 & .28728 \times 10^5 & .22435 \times 10^4 \\ -.58162 \times 10^5 & .12459 \times 10^5 & .97296 \times 10^3 \end{pmatrix}.$$

Then the sun's rectangular coordinates  $\bar{S}_\odot$ , in the RG system, are<sup>6</sup>

$$\bar{S}_\odot = A B \bar{E}_\odot.$$

3) The instantaneous satellite position is computed by subroutine POSIT.

Six quantities are sufficient to describe the position of a satellite, and SAO has chosen the following variables:

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<sup>6</sup>These calculations agree with those of the Nautical Almanac to three decimal places, which is sufficiently accurate for this purpose.



$\omega$ , argument of perigee,  
 $\Omega$ , right ascension of the ascending node,  
 $i$ , inclination,  
 $e$ , eccentricity,  
 $M$ , mean anomaly,  
 $n$ , mean motion.

By making these quantities dependent on time (i.e.,  $M = M(t)$ ,  $e = e(t)$ , etc.), the position of a satellite can be computed for any time. The time-dependent form of these quantities will be called the "elements." N.B. In this case,  $n = dM/dt$ . The format of the elements is the so-called DOI format.

a) The input elements are evaluated for time  $t$  by subroutine INST:

$$\begin{aligned}
 \omega &= \omega(t), \\
 \Omega &= \Omega(t), \\
 i &= i(t), \\
 e &= e(t), \\
 M &= M(t), \\
 n &= n(t).
 \end{aligned}$$

The semimajor axis is computed from

$$a = \left(\frac{K}{n^2}\right)^{\frac{1}{3}} \left[ 1 + \frac{1}{3} \frac{J}{p^2} \sqrt{1 - e^2} \left( -1 + \frac{3}{2} \sin^2 i \right) \right],$$

where

$$p = \left(\frac{K}{n^2}\right)^{\frac{1}{3}} (1 - e^2),$$

$$K = .753736886 \times 10^{+5} \text{ rev}^2 \text{ megameters}^3 \text{ day}^{-2},$$

$$J = .0660705456 \text{ megameters}.$$

b) Kepler's equation is solved by the method of iteration:

$$M = E - e \sin E \text{ for eccentric anomaly } (E).$$

c) The radius  $r$  is defined by

$$r = a(1 - e \cos E).$$

d) The true anomaly ( $v$ ) is determined by

$$\sin v = \frac{\sqrt{1 - e^2} \sin E}{1 - e \cos E},$$

and

$$\cos v = \frac{\cos E - e}{1 - e \cos E}.$$

Let

$$\xi = \Omega - \theta,$$

$$\eta = \omega + v.$$

Then the satellite's coordinates  $\bar{r}$  in the RG system are:

$$\bar{r} = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix},$$

where

$$y_1 = r(\cos \eta \cos \xi - \sin \eta \sin \xi \cos i),$$

$$y_2 = r(\cos \eta \sin \xi + \sin \eta \cos \xi \cos i),$$

$$y_3 = r(\sin \eta \sin i).$$

### Visibility criteria.

Let  $\bar{r}$ ,  $\bar{R}$ ,  $\bar{S}_\odot$  denote the satellite, station and sun vectors in the RG system. The satellite will be optically visible from  $\bar{R}$  if the  $\bar{r}$ ,  $\bar{R}$ ,  $\bar{S}_\odot$  configuration satisfies all the following criteria:

1) The satellite is visible if, and only if, it is outside the sun cone. The apex ( $\bar{A}$ ) of the cone is given by

$$\bar{A} = \frac{(r_e / \tan \chi) \bar{S}_\odot}{|\bar{S}_\odot|},$$

where  $r_e$  is the equatorial radius of the earth, and  $2\chi$  is the apex angle of the cone.

Then  $\mu$ , the angle between the satellite and the center line of the cone, is given by

$$\cos \mu = \frac{\bar{A} \cdot (\bar{A} - \bar{r})}{|\bar{A}| |\bar{A} - \bar{r}|} = \frac{\bar{A} \cdot (\bar{A} - \bar{r})}{(r_e / \tan \chi) |\bar{A} - \bar{r}|}.$$

The satellite is outside the cone and thus visible if:

$$\cos \mu < \cos \chi,$$

where  $\chi \cong 70'$  and  $r_e / \tan \chi \cong 313.2$  megameters.

2) The sun must be at least  $\alpha_{\min}$  degrees below the station horizon, i.e.,  $\alpha < \alpha_{\min}$ , where

$$\alpha = \cos^{-1} \left\{ \frac{\bar{R} \cdot (\bar{R} - \bar{S}_\odot)}{|\bar{R}| |\bar{R} - \bar{S}_\odot|} \right\}.$$

An input parameter,  $\alpha_{\min}$  may be different for each station.

3) The satellite must be at least  $\beta_{\min}$  degrees above the horizon, i.e.,  $\beta > \beta_{\min}$ , where

$$\beta = \cos^{-1} \left\{ \frac{\bar{R} \cdot (\bar{r} - \bar{R})}{|\bar{R}| |\bar{r} - \bar{R}|} \right\}.$$

An input parameter,  $\beta_{\min}$  may be different for each station.

## Great circle fitting

In the previous discussion we described the methods of evaluating the satellite's position ( $\bar{r}$ ) at time  $t$ , i.e.,  $\bar{r}(t)$ , and determining whether the satellite is visible for any, or all, of the observing stations. This section is concerned with the processing involved once a contiguous set of positions ( $\bar{r}_i$ ) has been determined to be observable from a station. Although any particular position may be observed by more than one station, the treatment assumes one set of  $n$  positions ( $\bar{r}_i$ ) constituting one pass across the sky and the observing station ( $\bar{R}$ ).

N.B. The positions  $\bar{r}_i$  are separated by equal intervals in time.

1) The positions  $\bar{r}_i$  are expressed in the local station coordinate system (hereafter referred to as the IS system) or the horizon system, and denoted by  $\bar{p}_i$ .

$$\bar{p}_i = CD(\bar{r}_i - \bar{R}) ,$$

where (as previously defined):

$\bar{r}_i$  is the satellite position in the RG system,

$\bar{R}$  is the station position in the RG system,

and

$$C = \begin{pmatrix} -\cos \lambda & \sin \lambda & 0 \\ \sin \lambda & \cos \lambda & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \lambda = \text{station longitude},$$

$$D = \begin{pmatrix} \sin \Phi & 0 & \cos \Phi \\ 0 & 1 & 0 \\ -\cos \Phi & 0 & \sin \Phi \end{pmatrix} \quad \Phi = \text{station latitude}.$$

2) The points ( $\bar{p}_i$ ) describe a path across the sky. It is possible by the method of least squares to determine a plane that passes through the observer and the points.

The angle swept out in this plane passing from the first point to the last point is called the great-circle arc, and the intersection of this plane with a unit sphere with origin at the station is called the great circle.

There are four options in the program for determining which points are selected for the great-circle arc used in a prediction:

- (1) two circles, one at each end;
- (2) one circle, about the center;
- (3) three circles (a combination of (1) and (2));
- (4) adjacent circles covering the whole arc.

3) Given the  $m$  points  $\bar{p}_i = (z_1^i, z_2^i, z_3^i)$ ,  $i = 1, 2, \dots, m$ , the least-squares great circle associated with these points is defined as the intersection of the least-squares plane,  $z_3 = az_1 + bz_2$ , with the unit sphere.  $a$  and  $b$  are determined from:

$$a = \left\{ \sum_i z_1^i z_3^i \sum_i (z_2^i)^2 - \sum_i z_1^i z_2^i \sum_i z_2^i z_3^i \right\} / \Delta, \quad i = 1, 2, \dots, m$$

$$b = \left\{ \sum_i z_2^i z_3^i \sum_i (z_1^i)^2 - \sum_i z_1^i z_2^i \sum_i z_1^i z_3^i \right\} / \Delta, \quad i = 1, 2, \dots, m$$

where

$$\Delta = \sum_i (z_1^i)^2 \sum_i (z_2^i)^2 - \left( \sum_i z_1^i z_2^i \right)^2, \quad i = 1, 2, \dots, m$$

It should be noted that this plane, together with the times of the first and last points used, completely define the camera settings, and, in particular, the culmination point. However, the culmination point calculated in this way is a fictitious one. Except for the second option, the culmination points need not have any relation to the closest point in the pass, which is the standard definition of culmination.

In practice, it is not necessary to track an object in a continuous arc for its whole pass; in fact, it is undesirable for the reasons given in the introduction. For these reasons three restrictions are placed on the arc to be photographed.

A slight digression is necessary here. Normally the object being tracked is quite faint in comparison with the star background which, incidentally, is used to determine the measured or observed position. Hence the cameras were built to follow the object and integrate the light. This means that a point image is necessary. With this technique the star images are trails and not points. The camera can also be used for stationary tracking (i.e., following the stars), when the satellite becomes a trail. However, the program described in this report is not constructed for this particular option.

The restrictions placed on the arc to be photographed are formulated as follows: The great circle is acceptable only if the image produced satisfies further conditions, i.e., the maximum vertical ( $v$ ) and horizontal ( $\psi$ ) displacements and the relative velocity ( $v_{rel}$ ) of the satellite with respect to the camera must be less than certain pre-specified constants.

To find the great circle containing the most points we take all the points, determine the plane defined by  $a$ ,  $b$ , calculate  $v$ ,  $\psi$ ,  $v_{rel}$  by the following procedure, and check these against the maximum values allowed. If the great circle is unacceptable, a point is discarded and the procedure is repeated until a satisfactory set of points is achieved. Clearly this is possible, since the case with only two points will give zero displacements. The points are discarded according to the option chosen; e.g., for option 1 the end point of the arc is always kept, and points on the culmination side only are discarded.

The following procedure is adopted:

Let

$$z_3^{i*} = az_1^i + bz_2^i \quad (\text{see fig. 1}).$$

Then the predicted point

$$\bar{p}_i^* = (z_1^i, z_2^i, z_3^{i*}),$$

the distance between  $\bar{p}_i$  and  $\bar{p}_i^*$  is

$$d_i = \frac{z_3^i - z_3^{i*}}{(a^2 + b^2 + 1)^{1/2}},$$

the normal to  $z_3 = az_1 + bz_2$  is

$$\bar{n} = \frac{1}{(a^2 + b^2 + 1)^{1/2}} \begin{pmatrix} a \\ b \\ -1 \end{pmatrix},$$

and  $\bar{p}'_1 = \bar{p}_1 - d_1 \bar{n}$  is the foot of the perpendicular dropped from  $\bar{p}_1$  onto the plane  $z_3 = az_1 + bz_2$ .

Then angle  $(\bar{p}_1, \bar{p}'_1)$  subtended at  $\bar{p}_1^*$  is

$$v_1 = \sin^{-1} \left\{ \frac{|\bar{p}_1 - \bar{p}'_1|}{|\bar{p}_1|} \right\},$$

angle  $(\bar{p}'_1, \bar{p}_1^*)$  subtended at the origin is

$$\psi_1 = \cos^{-1} \left\{ \frac{\bar{p}'_1 \cdot \bar{p}_1^*}{|\bar{p}'_1| |\bar{p}_1^*|} \right\},$$

and

$$v_{rel \ i} = \left\{ (v_1 - v_{1-l})^2 + (\psi_1 - \psi_{1-l})^2 \right\}^{1/2}$$

approximates the image displacement on successive frames.

For this great circle to be acceptable we must have:

$$v_1 \leq v_{\max},$$

$$\psi_1 \leq \psi_{\max}, \quad (\text{for all } i)$$

$$\frac{v_{rel \ i}}{v_{track}} \leq (M/.0014)^2,$$

where  $v_{\max}$ ,  $\psi_{\max}$ , and  $M$  (maximum drift in seconds of arc) are input parameters, and  $v_{\text{track}}$  is computed from

$$v_{\text{track}} = \cos^{-1} \left( \frac{\bar{p}_1 \cdot \bar{p}_m}{|\bar{p}_1| |\bar{p}_m|} \right) (m - 1)^{-1}.$$

$\bar{p}_1$  is the first point and  $\bar{p}_m$  is the last point used.

### Camera settings

We need to find the camera settings ( $h$ ,  $A$ ,  $TR_L$ ,  $TR_R$ ) implied by

- 1) The least-squares great circle defined by the intersection of  $z_3 = az_1 + bz_2$  and the unit sphere;
- 2)  $\bar{p}_1$ , the earliest point in the track; and
- 3)  $\bar{p}_m$ , the last point in the track.

The true altitude  $h_o$  and azimuth  $A_o$  of culmination are given by:

$$h_o = \tan^{-1} (a^2 + b^2)^{1/2},$$

$$\begin{cases} A'_o = \tan^{-1} \frac{a}{b}, & b \neq 0 \\ A'_o = 0, & b = 0 \end{cases}$$

$$\begin{cases} A_o = A'_o + \pi, & a < 0 \\ A_o = A'_o, & a \geq 0 \end{cases}$$

The camera altitude  $h$  and azimuth  $A$  are determined as follows ( $z_3$  is the zenith component in the IS system of the vector product of  $\bar{p}_1$  and  $\bar{p}_m$ ):

if  $z_3 \leq 0$ , then  $h = h_o$ , and  $A = A_o$ ;

if  $z_3 > 0$ , then  $h = \pi - h_o$ , and  $A = \pi + A_o$ .



The initial and terminal track angles  $TR_L$  and  $TR_R$  are computed as follows:

We define

$$\eta_L = \text{angle } (\bar{P}_L, \bar{C}) = \frac{\bar{P}_L \cdot \bar{C}}{|\bar{P}_L| |\bar{C}|},$$

$$\eta_R = \text{angle } (\bar{P}_R, \bar{C}) = \frac{\bar{P}_R \cdot \bar{C}}{|\bar{P}_R| |\bar{C}|},$$

$$\zeta = \text{angle } (\bar{P}_L, \bar{P}_R) = \frac{\bar{P}_L \cdot \bar{P}_R}{|\bar{P}_L| |\bar{P}_R|}.$$

$$\left. \begin{aligned} \text{If } \zeta = \eta_L - \eta_R, \text{ then } TR_L &= \pi/2 - \eta_L, \\ TR_R &= \pi/2 - \eta_R. \end{aligned} \right\}$$

$$\left. \begin{aligned} \text{If } \zeta = \eta_R - \eta_L, \text{ then } TR_L &= \pi/2 + \eta_L, \\ TR_R &= \pi/2 + \eta_R. \end{aligned} \right\}$$

$$\left. \begin{aligned} \text{If } \zeta = \eta_L + \eta_R, \text{ then } TR_L &= \pi/2 - \eta_L, \\ TR_R &= \pi/2 + \eta_R. \end{aligned} \right\}$$

Thereafter  $h$ ,  $A$ ,  $TR_R$ , the time of  $\bar{P}_L$ , and the time between  $\bar{P}_L$  and  $\bar{P}_R$  are retained for use in Phase 2.

## Conclusion

The accuracy of the field-reduced Baker-Nunn observations and the current state of orbital theory permits the calculation of predicted positions to sufficient accuracy for the Baker-Nunn camera to be operated completely by the 666 message. Of course the day-to-day problems of operating a tracking station do not allow this procedure to be used completely in practice. However, the character of the observing technique has changed because of this program. The rigid criterion of the image on the film has resulted in films more easily measured by the photoreduction section and more consistently good, precise reduced observations. This program allows coordination of all of the observations of all of the objects, thereby giving much better distribution of observations for scientific research. It allows the station to observe objects much closer to the horizon, far from culmination, which gives better orbital distribution to the observations for orbital analysis. The program has been operational for more than a year with very satisfactory results: Some of the field personnel have called its output "uncanny."

## Acknowledgments

We thank George Veis, Jack Slowey, and the many people at the Smithsonian Astrophysical Observatory who contributed ideas and valuable discussion to the formulation of this program. SCROGE could not have been written without their tireless efforts. We also thank Mrs. B. Berner and Miss Nancy January, who helped with the final checkout and perfection of the program. Above all, Miss Clara Munford must be thanked by both the authors and the readers, for only through her painstaking checking of the mathematics and editing of this report has its publication been completed.

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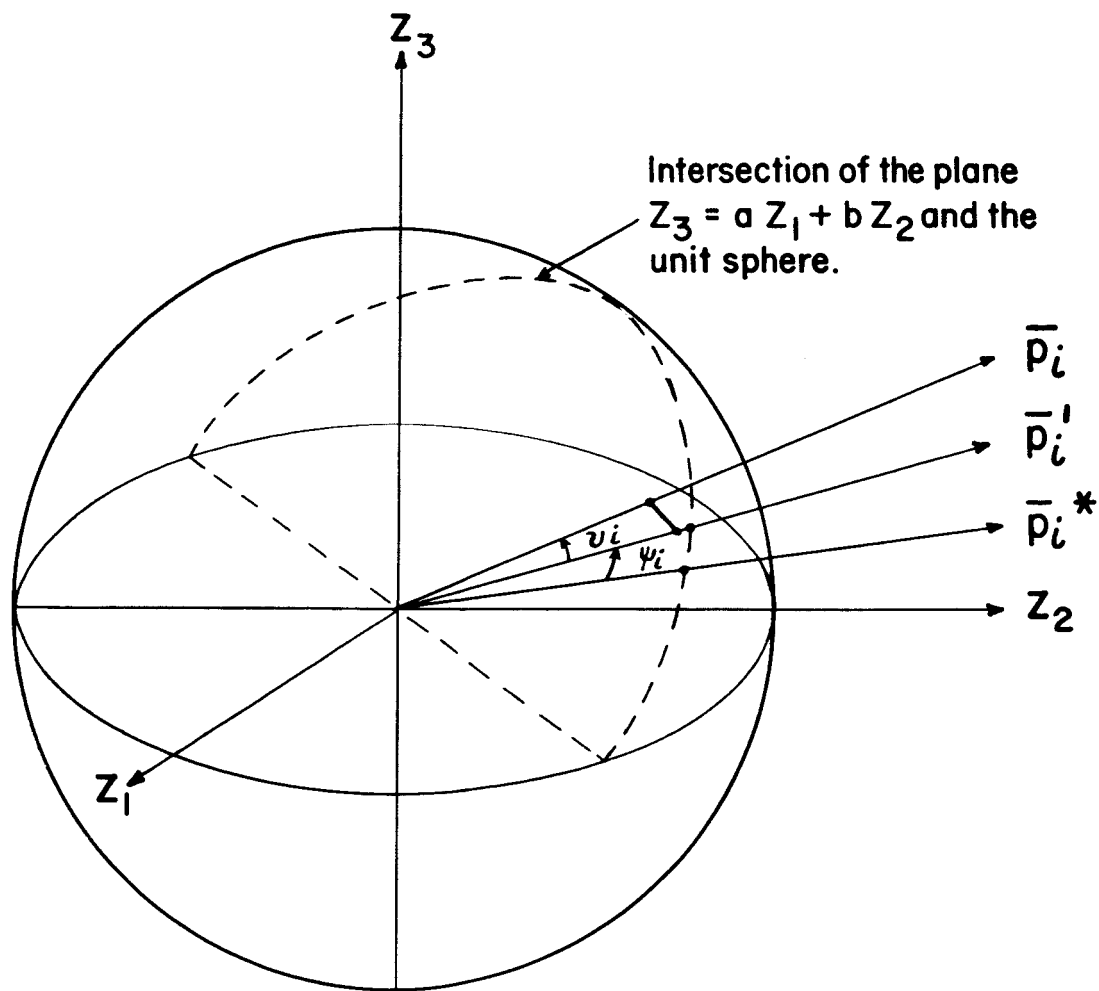


Figure 1

- $\bar{p}_i$  = the predicted direction
- $\bar{p}_i'$  = the foot of the perpendicular dropped from  $\bar{p}_i$  onto the plane  $z_3 = az_1 + bz_2$
- $\bar{p}_i^*$  = the direction of the camera
- $v_i$  = vertical displacement
- $\psi_i$  = horizontal displacement

ERRATUM

Special Report No. 105

The last equation on page 2 should read

$$e^2 = 1 - r^2 V^2 \cos^2 \theta / Gma$$

## NOTICE

This series of Special Reports was instituted under the supervision of Dr. F. L. Whipple, Director of the Astrophysical Observatory of the Smithsonian Institution, shortly after the launching of the first artificial earth satellite on October 4, 1957. Contributions come from the Staff of the Observatory. First issued to ensure the immediate dissemination of data for satellite tracking, the Reports have continued to provide a rapid distribution of catalogues of satellite observations, orbital information, and preliminary results of data analyses prior to formal publication in the appropriate journals.

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